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UNLIMITED CUMULATION UNDER ONE-DIMENSIONAL UNSTEADY COMPRESSION OF AN IDEAL GAS†

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Various methods of unlimited cumulation (UC) of an ideal (inviscid and non-heat-conducting) gas subject to one-dimensional unsteady compression by a plane, cylindrical, or spherical piston are considered. The most perfect method, namely, UC with isentropic compression from rest to rest, which is referred to as "ideal" (IUC), is compared with three other methods of UC, which correspond to well-known self-similar solutions of one-dimensional gas compression. The most effective of these is UC with a reflected shock wave, behind which the compressed gas is homogeneous and at rest, as in IUC. The efficiency of various methods of UC is estimated by the ratio of the work done during compression to the work in the case of IUC, the ratio of the internal energy to the total energy of the compressed gas, and the degree of gas homogeneity with respect to the Lagrangian variable. Computations of these characteristics are carried out for a perfect gas with various adiabatic exponents.

Interest in the problem of UC with one-dimensional compression has been stimulated by numerous applications, including controlled inertial thermonuclear fusion projects [1–4]. It is understood that only finite compression is realistic. Nevertheless, UC provides a fairly complete picture of compression to volumes that are small compared with the initial one, but non-zero. In a theoretical analysis it is natural to consider self-similar solutions. In the plane case one of these [5], consisting of a centred compression wave, yields in the limit an infinitely compressed gas with uniform distribution of the parameters with respect to the Lagrangian coordinate and with a small ratio of the internal energy to the total energy. Despite the fact that self-similar analogues of this solution for a cylindrical and a spherical piston have similar properties, it is to them that the main attention has been devoted [2–4, 6–8]. In the phase plane the integral curve describing this type of UC is a separatrix terminating at a saddle singular point.

The above solution and the corresponding UC, that is, UC1 do not exhaust all possible methods of UC that can be described by self-similar solutions. Two other self-similar solutions (more precisely, two other integral curves) provide homogeneous states of an infinitely compressed gas: one (UC2) with infinite speed directed towards the origin, that is, the volume centre, and the other one (UC3) with the gas remaining at rest behind the shock wave reflected at the centre. All these methods of unlimited compression, including IUC, involve the same finite compression time, equal to the time needed by a plane, cylindrical, of spherical sound wave to travel through the uncompressed gas. They also involve infinite work done by the piston. However, the work done in compressing the gas up to the same finite volume is different for the different ways of compression. The limits of the work ratios corresponding to UC are also different. As has already been mentioned, the most perfect method in unlimited compression is provided by IUC. We denote by A_0 the work done by the piston during IUC and by A_n the work done by the piston under UCN with $n = N = 1, 2, 3$. The ratios A_n/A_0 are found below for plane, cylindrical and spherical pistons compressing a perfect gas with various adiabatic exponents γ, and it is shown that $A_1/A_0 > A_2/A_0 > A_3/A_0. > 1$.

1. Let t be the time and let x be the Cartesian, cylindrical or spherical coordinate, namely, the distance to the "centre of compression", i.e. a plane, axis, or centre of symmetry of the volume containing some homogeneous gas initially at rest (in the plane case $x = 0$ either in the plane of symmetry or at the fixed wall). The piston begins to move from $x = x_i$ at time $t = t_i$. Here and below, unless otherwise stated, subscripts are assigned to parameters at the corresponding points. In symmetric compression the x-component $u \leqslant 0$ is the only non-zero component of the gas velocity. The 'null' subscript will be assigned to the initial parameters of the gas, the initial coordinate x_i of the piston will be taken as the length scale, and x_i/a_0, the time it takes for a plane, cylindrical, or spherical sound wave to travel across the

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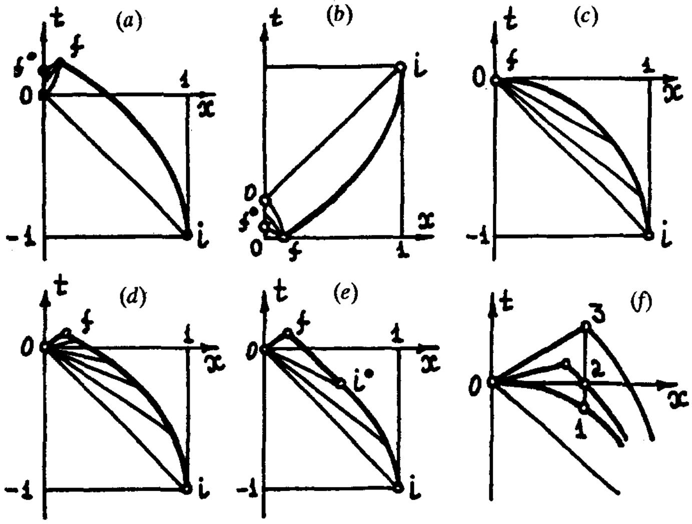

Fig. 1.

uncompressed gas will be taken as the time scale (a is the velocity of sound). The initial time t will be chosen in such a way that, as shown in Fig. 1, $x_i = 1$, $t_i = -1$ in the xt plane at the initial point i of the trajectory of the piston with the scaling adopted above. Thus, the origin $x = t = 0$ corresponds to the time of arrival on the t axis of a sound wave (C^--characteristic) starting from the initial point of the trajectory of the piston. In the solutions considered below the gas is at rest under the "initial" C^--characteristic: $x = -a_0, t = -t$ the gas is at rest while its parameters are constant and equal to their initial values (in the equation for the C^--characteristic a_0 disappears by the choice of the length and time scales).

Ideal UC can be obtained by taking the limit as $x_f \to 0$ of the solution [9] presented in Fig. 1(a), where if is the trajectory of the piston, io is the initial C^--characteristic, and off° is a packet of compression waves from the C^+-characteristics. Its rectilinear closing characteristic $f^\circ f$ is the lower bound of the domain of homogeneous compressed gas with $u \equiv 0$. The possibility of isentropic compression of finite duration from rest to rest follows practically at once from the invariance of the equations of one-dimensional unsteady gas dynamics with respect to the choice of the starting time t and by altering the signs of t and u simultaneously. As a result, the problem of compression from rest to rest can be obtained from the problem of gas expansion from rest to rest (Fig. 1b) and its solution can be reduced to computing the rarefaction wave packet $ff^\circ o$ and to determining the trajectory of the profile fi from the Goursat problem with data on the closing C^--characteristic fo of this packet and on the rectilinear C^+-characteristic oi. The gas parameters are constant on oi and $u \equiv 0$.

We will denote by ρ, e, h and s the density and the specific internal energy, enthalpy, and entropy of the gas. Since in the case of Fig. 1(a) the parameters of the compressed stationary gas are homogeneous, the following formulae for the work A done by the piston and the density ρ_f can be obtained from the laws of conservation of energy and mass, using the fact that the compression process is isentropic

$$\frac{A}{Me_0} = \frac{e_f}{e_0} - 1 \equiv e_f^\circ(\rho_f, \rho_0, s_0) - 1, \quad \rho_f = \rho_0 x_f^{-\nu} \tag{1.1}$$

Here M is the mass of compressed gas and $\nu = 1, 2, 3$ for a plane, cylindrical and spherical piston, respectively. For a perfect gas, by (1.1) we have

$$e_f^\circ \equiv e_f / e_0 = (\rho_f / \rho_0)^{\gamma-1} = x_f^{\nu(1-\gamma)} \tag{1.2}$$

The limit as $x_f \to 0$ in (1.1) and (1.2) yields infinitely increasing A and ρ_f, which correspond to UC. The parameters of the gas subject to unlimited compression other than $u \equiv 0$ and the velocity of the piston also increase to the right of f. As has already been observed, according to [10, 11], t_f tends to zero simultaneously and the total compression time approaches one. For IUC and other forms of UC studied below, the unlimited increase in the work done by the piston is a consequence of a non-integrable singularity of the pressure p as a function of x as $x \to x_f = 0$. Because of it, an infinitesimal (in the increment of x) section of the final trajectory makes an unlimited contribution to A. The same singularities of trajectories corresponding to different methods of UC are the main reason of their different effectiveness as measured by the aforementioned criteria. For any $x_f > 0$, preserving uniformly compressed gas at rest for $t > t_f$ requires the piston to stop abruptly, that is a physically unreal situation. However, as the piston continues its motion, the attained state is preserved everywhere under the trajectory of the shock wave, which travels from the point f towards the t axis in this case. This is important in applications, where x_f is always non-zero.

Restricting ourselves in what follows to a perfect gas with constant heat capacity, by [12] we can represent the flow parameters as follows when considering self-similar solutions

$$u = \frac{x}{t} U(\tau), \quad a^2 = \left(\frac{x}{t}\right)^2 \alpha(\tau), \quad \rho = \rho_0 R(\tau)$$
$$p = \rho_0 \left(\frac{x}{t}\right)^2 P(\tau), \quad P = \frac{\alpha}{\gamma} R, \quad \tau = \frac{a_0 t}{x} = \frac{t}{x} \tag{1.3}$$

where U, α and R are functions to be determined and τ is the self-similarity variable. By (1.3), for such self-similar solutions all parameters are constant along the rays $\tau = t/x = \text{const}$ emerging from the origin. Diagrams for two flows of this type are shown in Fig. 1(c) and (d). The first one corresponds to UC1, and the other one to bounded compression from rest to rest with the shock wave of, which leaves the gas moving towards the "centre". In the region behind the shock wave ($\tau_f < \tau \leq \infty$)

$$U \equiv 0, \quad R \equiv R_{f+}, \quad \alpha = \alpha_{f+}(\tau / \tau_f)^2$$

Here and henceforth the subscripts $f-$ and $f+$ are assigned to the parameters before and after the shock wave. In both cases the gas is stationary also under the initial C^--characteristic io, where $-\infty \leq \tau \leq -1$, $U \equiv 0$, $R \equiv 1$ and $\alpha \equiv \tau$. The last equality is the result of choosing the velocity of sound in the uncompressed gas as the scale of u and a, which is consistent with the length and time scales introduced earlier. On io

$$\tau = -1, \quad U = 0, \quad \alpha = 1, \quad R = 1, \quad P = 1/\gamma \tag{1.4}$$

Compression is isentropic in the case of Fig. 1(c) for all τ and in the case of Fig. 1(d) for $\tau < \tau_f$. Thus, for such τ

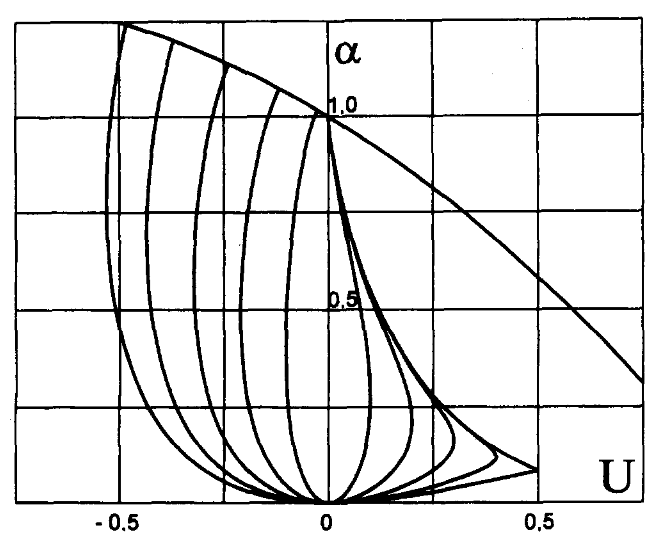

Fig. 2.

$$R=\left(\frac{\alpha}{\tau^2}\right)^{1/(\gamma-1)} \tag{1.5}$$

Taking (1.5) into account, the construction of self-similar solutions of the type under consideration can be reduced to the integration of the two equations

$$\frac{dU}{d\alpha}=\frac{U[\nu\alpha-(U-1)^2]}{2\alpha[\alpha-(U-1)(U-1+nU)]},\quad \frac{d\tau}{d\alpha}=\frac{\tau[\alpha-(U-1)^2]}{2\alpha[\alpha-(U-1)(U-1+nU)]} \tag{1.6}$$
$$n=(\nu-1)(\gamma-1)/2$$

with initial conditions (1.4). These equations are the same, apart from the notation, as those in [12]. The above equalities describing a gas at rest naturally satisfy these equations.

The first equation in (1.6), which does not contain τ, has three singular points, which are important in what follows: the node $U = 0$, $\alpha = 1$, which by (1.4) corresponds to the initial C^-characteristic *io*, the node $U = \alpha = 0$, and the saddle

$$U=\frac{2}{2+\nu(\gamma-1)},\quad \alpha=\frac{\nu(\gamma-1)^2}{[2+\nu(\gamma-1)]^2} \tag{1.7}$$

One of the separatrices of the saddle starts at the first node. The other separatrix, having passed through the second node, reaches the "shock wave line", i.e. the parabola [12]

$$\alpha=(1-U)\left(1+\frac{\gamma-1}{2}U\right) \tag{1.8}$$

for all ν. This parabola, the sections of the separatrices found numerically for $\nu = 3$ and $\gamma = 5/3$, and several integral curves of the equation under consideration are shown in Fig. 2. In the case of a solution with a shock wave (Fig. 1d) the points of the parabola (1.8) with $U < 0$ lying to the right of the intersection of the parabola and the second separatrix correspond to reducing the gas velocity from $U_{f-} < 0$ in front of the shock wave up to rest behind the wave. By (1.3), (1.8) and the relationships on the shock wave, the parameters of the gas at rest immediately behind the wave have the following values

$$U_{f+}=0,\quad \alpha_{f+}=1-\frac{\gamma+1}{2}U_{f-},\quad R_{f+}=R_{f-}(1-U_{f-})$$

In the plane case ($\nu = 1$, $n = 0$) all the expressions in square brackets in (1.6) are equal to $\alpha - (U-1)^2$. Thus, after reduction, in place of (1.6) we obtain the system

$$\frac{dU}{d\alpha}=\frac{U}{2\alpha},\quad \frac{d\tau}{d\alpha}=\frac{\tau}{2\alpha} \tag{1.9}$$

the integration of which yields (k_i being constants): $U = k_1\tau$, $\alpha = k_2\tau^2$, that is, a uniform flow with $u \equiv k_1$ by (1.3). The expressions in brackets which have been cancelled, may vanish themselves. This possibility yields a "sonic parabola": $\alpha = (U-1)^2$, which describes a centred compression wave with rectilinear C^--characteristics, focusing at the origin $x = t = 0$. As a result, we obtain the well-known self-similar solution (Fig. 1e) with constant parameters in the triangle $i°$ of between the centred compression wave and the shock wave. The corresponding Riemann invariant is preserved in the centred wave [5, 13]. This condition, along with (1.3) and the equation of the sonic parabola, yields

$$\tau=\frac{\gamma-1}{2}+\frac{\gamma+1}{2}(U-1),\quad \alpha=(U-1)^2$$

The resulting values of U and α corresponding to UC ($u \to -\infty$, $a \to \infty$, $\tau \to -0$) are the same as those U and α that yield (1.7) for $\nu = 1$. This may appear curious at first. Indeed, for $\nu = 1$ the self-similar solution in the plane $U\alpha$ can be described either by the finite relationship $\alpha = (U-1)^2$, or by the first differential equations of system (1.9) with one singular point, namely, the node $U = \alpha = 0$. On the other hand, formulae (1.7) yield the coordinates of an entirely different singular point, a saddle, which is not present when $\nu = 1$. A possible solution of this paradox may lie in the change from a discrete to a continuous parameter ν in (1.6) followed by letting ν tend to unity. In this case the first equation in (1.6) has a saddle for all ν, and its coordinates are defined by (1.7), including the case when $\nu = 1$. In the plane case the second separatrix is the parabola

$$\alpha = (\gamma - 1)^2 U^2 / 4$$

whose intersection with the shock-wave line (1.8) yields

$$U_{f-} = \frac{\gamma - 3 - \sqrt{(3-\gamma)^2 + 4(\gamma^2 - 1)}}{\gamma^2 - 1} \tag{1.10}$$

For the self-similar solutions under consideration, to determine the work done by the piston we can use the integral laws of concentration of mass and energy, which for any closed contour Γ in the xt plane have the form

$$\oint_\Gamma \rho x^{\nu-1}(dx - udt) = 0, \quad \oint_\Gamma \rho x^{\nu-1}\left[\left(e + \frac{u^2}{2}\right)dx - \left(h + \frac{u^2}{2}\right)udt\right] = 0 \tag{1.11}$$

$$e = \frac{a^2}{\gamma(\gamma-1)}, \quad h = e + \frac{p}{\rho} = \frac{a^2}{\gamma - 1}$$

As Γ we take the contour formed by the initial characteristic oi, on which $\tau = -1$, $U = 0$ and $R = \alpha = 1$, by an arbitrary ray $\tau = \text{const} > -1$, and by the trajectory of the piston, where $dx/dt = u$ by the no-flow condition. Then from (1.11), using (1.3) and the fact that $x_i = 1$, we obtain

$$x^\nu R(1-U) = x^\nu \left(\frac{\alpha}{\tau^2}\right)^{1/(\gamma-1)}(1-U) = 1 \tag{1.12}$$

$$\frac{A}{Me_0} = \frac{2\alpha(1-\gamma U) + \gamma(\gamma-1)(1-U)U^2}{2\tau^2(1-U)} - 1$$

Here $x = x(\tau)$ and $A = A(\tau)$ is the coordinate and the work done by the piston for any $\tau \geqslant -1$, the first formula with the 'second' expression on the left-hand side in the case of Fig. 1(d) and (e) being valid only under the shock wave.

2. The formulae presented above enable us to compare various methods of UC. Along with IUC (Fig. 1a with $x_f \to 0$) and UC1 (Fig. 1c), we shall consider two more forms of UC, namely, UC and UC3. The first of these can be obtained as a limit (when $x_f \to 0$) of states corresponding to a node at the origin and $\tau = 0$ lying on the integral curves in Fig. 2, and to segments δ of the x axis in Fig. 1(d), that is, to the motion of homogeneous gas towards the centre. Another possibility of unlimited compression (UC3) can be obtained as the same limit (when $x_f \to 0$) of gas states after the passage of the shock wave of (Fig. 1d and e). First, the work ratios will be compared for the above methods of UC. Initially, the ratios can be found for compression up to the same finite x_f, followed by passing to the limit as $x_f \to 0$. The method of comparison is explained in Fig. 1(f) showing the terminal sections of the corresponding trajectories of the piston along with the trajectories of the shock waves and the initial characteristic oi. The x coordinates of the points 1, 2, 3 are the same. For any "self-similar" compression, from the first formula in (1.12) we have

$$\tau^2 = x^{\nu(\gamma-1)}\alpha(1-U)^{\gamma-1}$$

Hence, from (1.2) and the second formula in (1.12) we find that for identical small xs the ratio of the work A done in self-similar compression to the work A_0 in ideal isentropic compression is equal to

$$\frac{A}{A_0} \approx \frac{2(1-\gamma U) + \gamma(\gamma-1)(1-U)U^2/\alpha}{2(1-U)^\gamma}$$

As $x_f \to 0$ this formula becomes precise. Substituting into this formula U and α from (1.7) for UC1, $U(0) = 0$ and $U^2(0)/\alpha(0) = K \neq 0$ for UC2, and expressing α_{f-} in terms of U_{f-}, from (1.8) for UC3 we find that

Table 1

	ω_1			ω_2			ω_3		
γ	$\nu = 1$	2	3	1	2	3	1	2	3
1.01	211	105	69.6	202	97.0	62.7	192	87.2	53.4
1.05	48.4	23.5	15.4	42.0	18.6	11.4	34.8	12.7	6.29
1.1	27.5	13.0	8.40	22.0	9.10	5.40	16.1	4.77	1.92
1.2	16.8	7.59	4.81	12.0	4.60	2.64	7.29	1.56	0.42
1.3	13.1	5.73	3.58	8.67	3.15	1.78	4.56	0.75	0.15
1.4	11.3	4.78	2.95	7.00	2.46	1.38	3.27	0.43	0.07
$^5/_3$	9.10	3.61	2.17	5.00	1.65	0.92	1.85	0.16	0.02
3	7.00	2.37	1.37	3.00	0.90	0.51	0.66	0.03	0.001

$$
\begin{aligned}
\omega_1 &\equiv \frac{A_1}{A_0} - 1 = \left[1 + \frac{2}{\nu(\gamma-1)}\right]^{\gamma} - 1, \quad \omega_2 \equiv \frac{A_2}{A_0} - 1 = \frac{\gamma}{2}(\gamma-1)K \\
\omega_3 &\equiv \frac{A_3}{A_0} - 1 = \frac{2-(\gamma+1)U_{f-}}{(1-U_{f-})^{\gamma}[2+(\gamma-1)U_{f-}]} - 1, \quad K = \frac{U^2(0)}{\alpha(0)}
\end{aligned} \tag{2.1}
$$

In the plane case U_{f-} is given by (1.10), $K = 4/(\gamma-1)^2$, and $\omega_2 = 2\gamma/(\gamma-1)$. An equivalent formula was obtained earlier in [9]. For a cylindrical piston and a spherical piston K and U_{f-} can be found in the course of a numerical construction of the separatrix leading from the saddle (1.7) through the node $U = \alpha = 0$ towards the intersection with the shock-wave line (1.8). The results of computations carried out for various γ and ν are collected in Table 1. Moreover, explicit formulae for $\nu = 1$ and an analysis of numerical results for $\nu \neq 1$ indicate that $\omega_1 \to \omega_2 \to \omega_3 \to \infty$ and $A_1/A_2 \to A_2/A_3 \to 1$ for all ν and $\gamma \to 1$. As follows from Table 1 and from the position of the piston trajectories with respect to one another in Fig. 1(f) (at point 1 the velocity of the piston and, consequently, the pressure are greater than at point 2, and so on): $\omega_1 > \omega_2 > \omega_3$ for any fixed ν and $\gamma > 1$. As ν and γ increase, each ω_i decreases. If γ is far from unity, then ω_3 is only slightly greater than zero in the case of a spherical piston. Since the gas behind the reflected shock wave is homogeneous and at rest in this case, that is, the entire work done by the piston is used to increase its internal energy, all the characteristics of spherical UC3 and IUC that are being compared are close to one another in such cases. Similarly, the characteristics can be close to one another as a result of "self-deceleration" of gas moving towards the centre. As γ decreases and $\nu = 3$ becomes $\nu = 2$, the effect of self-deceleration weakens and totally vanishes for $\nu = 1$. However, for all ν the pressure drop on the shock wave is finite and decreases as ν and γ increase.

UC1 and UC2 are significantly inferior to UC3, not only because of the work expended in compression. In the case of UC1 the compressed gas is extremely inhomogeneous: for $t = t_1 < 0$ as small as desired its properties can change from those of an uncompressed gas in a small neighbourhood of the t axis to extremely large velocities, densities, pressures, etc. near the piston. We introduce the Lagrangian coordinate m marking the particles by the equality

$$
m = \frac{1}{x_i^{\nu}\rho_0}\int_0^x \rho\, dx^{\nu} = \int_0^x R(\tau)\, dx^{\nu}
$$

where the integral is taken at $t = t_1$, so that $\tau = t_1/x$. By the law of conservation of mass $0 \leqslant m \leqslant 1$. The value $-1 \leqslant \tau < \tau_p$ corresponds to the gas compression by a piston as $t = t_1 \to 0$, where $\tau = \tau_p \to 0$ on the trajectory of the piston. In this case, by (1.3), (1.5) and (1.7)

$$
\frac{u_p}{a_0} \approx \frac{2}{\tau_p[2+\nu(\gamma-1)]}, \quad \frac{a_p^2}{a_0^2} \approx \frac{\nu(\gamma-1)^2}{\tau_p^2[2+\nu(\gamma-1)]^2}, \quad \frac{\rho_p}{\rho_0} = \left(\frac{a_p}{a_0}\right)^{2/(\gamma-1)} \tag{2.2}
$$

on the piston for the method of compression under consideration.

Furthermore, it can be shown that almost all the gas is concentrated near the piston as t_1 and $\tau_p \to 0$, where m and τ are related by

Table 2

	$\eta_1 \times 10^2$			$\eta_2 \times 10^2$		
γ	$\nu = 1$	2	3	1	2	3
1.01	0.49	0.98	1.46	0.49	1.02	1.57
1.05	2.33	4.50	6.67	2.33	5.11	8.10
1.1	4.35	8.33	12.0	4.35	9.87	15.6
1.2	7.69	14.3	20.0	7.69	17.9	27.5
1.3	10.3	18.7	25.7	10.3	24.1	35.9
1.4	12.5	22.2	30.0	12.5	28.9	42.0
$^5/_3$	16.7	28.6	37.5	16.7	37.7	52.0
3	25.0	40.0	50.0	25.0	52.6	66.3

$$m = \left(\frac{\tau_p}{\tau}\right)^l, \quad l = \frac{2+\nu(\gamma-1)}{\gamma-1} \tag{2.3}$$

and the distribution of the parameters over the particles, i.e. the dependence of the gas properties on m, is given by

$$\left(\frac{u}{u_p}\right)^2 \approx \left(\frac{a}{a_p}\right)^2 = \left(\frac{e}{e_p}\right)^2 = \left(\frac{\rho}{\rho_p}\right)^{(\gamma-1)} \approx m^{2/l} \tag{2.4}$$

As $\tau_1 \to 0$ formulae (2.4) become precise, demonstrating that infinitely compressed gas in inhomogeneous for UC1.

Finally, for UC1 and UC2, unlike IUC and UC3, as a rule only a small part of the work done by the piston contributes to increasing the internal energy of the compressed gas. By (2.2)–(2.4), for UC1 the ratio η_1 of the internal energy of infinitely compressed gas to its total energy ("the work efficiency coefficient") is equal to

$$\eta_1 = (\gamma-1)\left(\frac{2+\nu}{\nu}\gamma - 1\right)^{-1}$$

For UC2 in the case of the motion of homogeneous gas towards the "centre" has analogous coefficient is given by

$$\eta_{2'} = 1/(\omega_2 + 1)$$

with ω_2 from (2.1). In the case of a plane piston $\eta_2 = \eta_1$.

Values of $n_i \times 10^2$, computed for various ν and γ, are collected in Table 2. Moreover, $\eta_1 \to \eta_2 \to 0$ as $\gamma \to 1$. For $\nu \neq 1$ and for all γ UC2 is somewhat more efficient than UC1 as far as η is concerned, while each of these two compression methods is inferior in this respect to IUC and UC3, for which $\eta = 1$.

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